Exercise sheet for "Einführung in das Rechnergestützte Arbeiten"

Abstract

In this exercise all the tools introduced in the lecture should be combined.
The goal is to write a LaTeX document in which the solution of physics problems are presented. The prepared document including all source files (tex-source, svg-file, maple worksheet and jupyter notebook) has to be presented to a tutor in the final exercise.

The solutions can be certified on Wednesday 20.4.2022, 2:00-3:30 pm.

1 **LaTeX** Generate a LaTeXdocument of the class `article`. This document should have a titlepage with your name. All further content will be given by the problems given below. The solutions of these problems should be included in the document as described there.

2 **Inkscape**

The double pendulum is subject of classical theoretical physics II. There, the equations of motion are derived, and solved analytically only for small displacements.

Here we will study the general case, in which the double pendulum can show chaotic behaviour. The equations of motion can be derived analytically but only solved numerical.

Draw with Inkscape, as suggested in figure 1, a sketch of the system in which all relevant variables are shown.
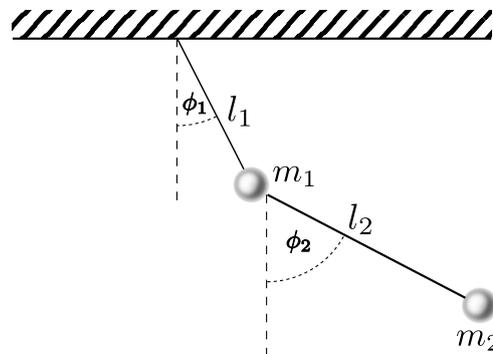

Figure 1: Sketch of the double pendulum

Save the sketch as PDF file and include the figure in your document. In the document the figure should have a short descriptive caption.

Hint: Greek letters can either be achieved with the LaTeX-plugin or be entered as unicode text. For the latter, while in text input press CTRL-u to change to unicode mode and enter the character code (`http://unicode.org/charts/PDF/U0370.pdf`). (E.g. the character code of ϕ is 03D5.)

3 **Maple**

The equations of motion for the displacement angles ϕ_1 and ϕ_2 of the double pendulum arise from the Lagrange equation as

$$l_1\ddot{\phi}_1(m_1 + m_2) + m_2 l_2 \ddot{\phi}_2 \cos(\phi_1 - \phi_2) + m_2 l_2 \dot{\phi}_2^2 \sin(\phi_1 - \phi_2) + g\sin(\phi_1)(m_1 + m_2) = 0,$$

$$m_2 l_1 \cos(\phi_1 - \phi_2)\ddot{\phi}_1 + l_2 m_2 \ddot{\phi}_2 - m_2 l_1 \sin(\phi_1 - \phi_2)\dot{\phi}_1^2 + m_2 g \sin(\phi_2) = 0. \qquad (1)$$

a) The system of differential equations (1) can only be solved numerically. To that end all constants must be assigned numerical values. Choose

$$m_1 = 1, \qquad m_2 = 2, \qquad l_1 = 1, \qquad l_2 = 2, \qquad g = 1$$

and plot with the boundary conditions

$$\phi_1(0) = 1.50, \qquad \phi_2(0) = 2, \qquad \dot{\phi}_1(0) = 0, \qquad \dot{\phi}_2(0) = 0$$

the evolution of $\phi_1(t)$ and $\phi_2(t)$ in the time interval $t = 0 \ldots 100$, combine both curves in one graph.

Tips:

- *Don't forget to load the package* `plots` *beforehand.*
- *If Maple is not instructed explicitly to solve the system of equations numerically as an option, it will start a futile attempt to find an analytic solution. . . for a long time. . . a very long time. . .*

b) Create another combined plot in which you repeat the calculation with changed initial condition for $\phi_1(0) = 1.502$.

Remark: As you will see, the system reacts sensitively to small changes in the initial conditions. This is a characteristic property of chaotic systems.

c) Export the two combined plots and include them into them into your document with a short descriptive text.

d) Amend your document with a short descriptive text including the above equations and initial conditions typeset in LaTeX[1].

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[1] Manually! The LaTeX export feature of Maple should not be used.

4 Python/Visualization

In the lab course experiment "Determination of e/m of the electron" the specific charge of the electron is measured with different methods. Following the method of Busch, electrons are accelerated in a cathode ray tube by a voltage U and deflected on a spiral trajectore by electro magnetic fields. From the dependence of the spiral radius on the voltage U and the magnetic field $\mathbf{B}$ (created by a current I) it is possible to calcualte the specific charge. In the experiment U is adjusted and I is the measured quantity. I gemessen.

Without going into the details of the experiment and futher derivations, the formula

$$U = \eta \frac{e}{m} \mu_0^2 I^2 \qquad \eta = 2.36 \cdot 10^5 \text{ and } \mu_0 = 4\pi \cdot 10^{-7} \text{N A}^{-2} \tag{2}$$

is given. Here η is a dimensionless quantity that is determined by the coil geometry and the experimental setup and μ_0 the vacuum permeability.

Exercise:

The file `messwerte.dat` contains a fictional table of measured coil current I and acceleration voltage U. Create a Jupyter notebook with the following steps and document each with a markdown cell.

a) Load the measurement data from the into two variables `U` and `I`.

Plot I over U and label the axes.

b) Calculate (in one step) for all voltages e/m and then take the average.

c) Determine e/m by fitting a function to the data: Create a function $U(I)$ given by

$$U = c \cdot I^2 \tag{3}$$

and perform a numerical fit to the data. Extract the constant c and its standard derivation Δc. In a last step, calculate e/m from $\frac{e}{m} = c/(\eta \mu_0^2)$ and the standard derivation from $\Delta \frac{e}{m} = \Delta c/(\eta \mu_0^2)$.

d) Plot the data and the fitted curve in the same graph. Put the current I on the x axis and the voltage U on the y axis. Supplement the graph with axes labels and a legend and export it as an image.

e) Import the file into your document and add a short caption.

Describe in the document which value you got for the specific charge e/m and the standard derivation (no error propagation needed).

f*) (advanced, optional) How do you have to change the call to `curve_fit` such that e/m can be calculated successfully directly from Eq. 2 without taking a detour through c? What result do you get?